Nonrelativistic Nambu-Goldstone Modes Associated with Spontaneously Broken Space-Time and Internal Symmetries

Michikazu Kobayashi[1] and Muneto Nitta[2]
[1]*Department of Physics, Kyoto University, Oiwake-cho, Kitashirakawa, Sakyo-ku, Kyoto 606-8502, Japan*
[2]*Department of Physics and Research and Education Center for Natural Sciences, Keio University, Hiyoshi 4-1-1, Yokohama, Kanagawa 223-8521, Japan*

(Received 19 March 2014; revised manuscript received 6 August 2014; published 18 September 2014)

We show that a momentum operator of a translational symmetry may not commute with an internal symmetry operator in the presence of a topological soliton in nonrelativistic theories. As a striking consequence, there appears a coupled Nambu-Goldstone mode with a quadratic dispersion consisting of translational and internal zero modes in the vicinity of a domain wall in an O(3) σ model, a magnetic domain wall in ferromagnets with an easy axis.

DOI: 10.1103/PhysRevLett.113.120403 PACS numbers: 05.30.Jp, 03.75.Lm, 03.75.Mn, 11.27.+d

Introduction.—Symmetry is one of the most important guiding principles in describing nature in quantum physics. In particular, relativistic quantum field theories rely on symmetry principle and have been quite successful in unifying fundamental forces. Gauge symmetries of electromagnetic, weak, and strong interactions can be unified to one group in grand unified theories. However, space-time symmetry related to gravity could not be, because of the Coleman-Mandula theorem [1] showing that internal symmetry and space-time symmetry must be a direct product [2]. These are all based on relativistic field theories.

In this Letter, we find a symmetry algebra including both the space-time symmetry and an internal symmetry in *nonrelativistic* field theory,

$$[P, \Theta] = W \neq 0, \tag{1}$$

where P is a translational operator, Θ is an internal symmetry operator, and W is a "central" extension. We show that the commutation relation, Eq. (1), is possible in the presence of a topological soliton in nonrelativistic theories. In a practical model, the central charge W is a topological charge of a domain wall [3]. The central charge W vanishes in the corresponding relativistic model. As a consequence of our novel algebra, a Nambu-Goldstone (NG) mode for the translational symmetry (a ripple mode or ripplon if quantized) is coupled to that for the internal U(1) symmetry (a magnon) when the presence of a domain wall breaks both the symmetries; They give rise to one NG mode with a quadratic dispersion relation, although two symmetry generators are spontaneously broken. This is in contrast to the corresponding relativistic model, in which these two modes, ripplon and magnon, appear independently with linear dispersion relations, which corresponds to the fact that W vanishes. This phenomenon itself is already known as type-II or type-B NG modes in nonrelativistic theories [4–8]. However, previously known examples are either NG modes of both internal symmetries such as ferromagnets or those of both space-time symmetry such as vortices and lumps (Skyrmions) [9,10], while our case mixes them together because of Eq. (1). We derive the dispersion relation in two approaches: the effective field theory for topological solitons and Bogoliubov theory. We also study a model interpolating between relativistic and nonrelativistic theories and find unexpectedly a coupled NG mode in an interpolating region even though it has the Lorentz invariance.

Models and a domain wall.—We start from the following relativistic, and nonrelativistic $\mathbb{C}P^1$ Lagrangian densities $\mathcal{L}_{\rm rel}$ and $\mathcal{L}_{\rm nrel}$ with an Ising-type potential,

$$\mathcal{L}_{\rm rel} = \frac{|\dot{u}|^2 - |\nabla u|^2 - m^2|u|^2}{(1+|u|^2)^2},$$
$$\mathcal{L}_{\rm nrel} = \frac{i(u^*\dot{u} - \dot{u}^*u)}{2(1+|u|^2)} - \frac{|\nabla u|^2 + m^2|u|^2}{(1+|u|^2)^2}, \tag{2}$$

where $u \in \mathbb{C}$ is the complex projective coordinate defined as $\phi^T = (1, u)^T/\sqrt{1+|u|^2}$ with normalized two scalar fields $\phi = (\phi_1, \phi_2)^T$. $\mathcal{L}_{\rm rel}$ and $\mathcal{L}_{\rm nrel}$ are equivalent to O(3) nonlinear σ models,

$$\mathcal{L}_{\rm rel} = \frac{1}{4}\{|\dot{\boldsymbol{n}}|^2 - |\nabla \boldsymbol{n}|^2 - m^2(1-n_3^2)\},$$
$$\mathcal{L}_{\rm nrel} = \frac{\dot{n}_1 n_2 - n_1 \dot{n}_2}{2(1+n_3)} - \frac{1}{4}\{|\nabla \boldsymbol{n}|^2 + m^2(1-n_3^2)\}, \tag{3}$$

under the Hopf map for a three-vector of scalar fields $\boldsymbol{n} \equiv \phi^\dagger \boldsymbol{\sigma} \phi$ with the Pauli matrices $\boldsymbol{\sigma}$. These models describe ferromagnets with one easy axis.

A Lagrangian density interpolating between $\mathcal{L}_{\rm rel}$ and $\mathcal{L}_{\rm nrel}$ is given as the following form:

$$\mathcal{L}_G = \phi_0^2\left\{\frac{|\dot{u}|^2}{c^2(1+|u|^2)^2} - \frac{|\nabla u|^2}{(1+|u|^2)^2} + \frac{iM(u^*\dot{u} - \dot{u}^*u)}{\hbar(1+|u|^2)} - \frac{m^2|u|^2}{(1+|u|^2)^2}\right\}, \tag{4}$$

where $\phi_0^2(>0)$ is a real positive (decay) constant having the dimension of [energy]/[length]. In the following, we omit ϕ_0^2 by measuring $\mathcal{L}_G$ in the unit of ϕ_0^2: $\mathcal{L}_G \to \phi_0^2\mathcal{L}_G$. A detailed derivation of the Lagrangian density $\mathcal{L}_G$ is discussed in the Supplemental Material [11]. $\mathcal{L}_{\rm rel}$ is obtained as the massless limit $M \to 0$ of $\mathcal{L}_G$, while $\mathcal{L}_{\rm nrel}$ is the nonrelativistic limit $c \to \infty$ of $\mathcal{L}_G$ [12].

The actions $S = \int d^4x\mathcal{L}_G$ are invariant under a global discrete $\mathbb{Z}_2$ transformation: $u \leftrightarrow 1/u^*$, a global U(1) phase rotation: $u \to ue^{i\alpha}$. S is also invariant under the Poincaré transformation as long as c is positive finite, or the Galilean transformation in the nonrelativistic limit $c \to \infty$, as shown in the Supplemental Material [11]. There are two discrete vacua $|u| = 0$ or $|u| \to \infty$, and $m > 0$ is defined as the energy gap between them. For these vacua, the $\mathbb{Z}_2$ symmetry for the global discrete transformation is spontaneously broken. In the framework of the nonlinear σ models, Eq. (3), the vacua are expressed as $n_1 = n_2 = 0$, $n_3 = \pm 1$, and the last $m^2(1-n_3^2)$ terms are regarded as the Ising potential.

Dynamics of u can be obtained by the Euler-Lagrange equation for $\mathcal{L}_G$,

$$\frac{(1+|u|^2)\ddot{u} - 2u^*\dot{u}^2}{c^2} - \frac{2iM(1+|u|^2)\dot{u}}{\hbar} = (1+|u|^2)\nabla^2 u - 2u^*(\nabla u)^2 - m^2(1-|u|^2)u. \quad (5)$$

We next consider a static domain- or antidomain-wall solution interpolating the two vacua. The flat and static domain-wall solution perpendicular to the z axis is [13] (see the Supplemental Material [11])

$$u_0 = \exp\{m(z - Z) + i\alpha\}, \quad (6)$$

where α $(0 \le \alpha < 2\pi)$ and $Z \in \mathbb{R}$ are phase and translational moduli of the domain wall. This is known as a magnetic domain wall in ferromagnets with an easy axis.

In the presence of the domain wall, the $H_1 \simeq \mathrm{U}(1) \times \mathbb{R}^3$ symmetry is further spontaneously broken, where U(1) is the global symmetry for the internal phase rotation and $\mathbb{R}^3$ is the three-dimensional translational symmetry in a space. The remaining symmetry is $H_2 \simeq \mathbb{R}^2_{xy}$, where $\mathbb{R}^2_{xy}$ indicates the translation along the xy plane [14]. Breaking symmetries $H_1/H_2 \simeq \mathrm{U}(1) \times \mathbb{R}_z$ due to the domain wall are the internal U(1) phase rotation and translation along the z direction, and two moduli α and Z in Eq. (6) are regarded as corresponding NG modes in the vicinity of the domain wall. The NG mode α is the phase mode known as a magnon localized in the domain wall. The other NG mode for Z is the translational surface mode of the domain wall, known as a ripple mode, or ripplon if quantized, in condensed matter physics. In the following, we show that the localized magnon and ripplon are coupled to each other to become one "coupled ripplon" mode with fixed dispersion relation and amplitude.

Low-energy effective theory of a domain wall.—We next consider the NG modes excited along the domain wall by constructing the effective theory on a domain wall by the moduli approximation [15]. Introducing $\boldsymbol{r} = (x, y)$, and t dependences of two moduli α and Z as $\alpha(\boldsymbol{r}, t)$ and $Z(\boldsymbol{r}, t)$, we consider the ansatz u as

$$u = \exp[m\{z - Z(\boldsymbol{r}, t)\} + i\alpha(\boldsymbol{r}, t)]. \quad (7)$$

Inserting Eq. (7) to Eq. (4), the effective Lagrangian $L_G^{\rm eff}$ defined as $L_G^{\rm eff} = \lim_{L\to\infty}\int_{-L}^{L} dz\mathcal{L}_G$ becomes

$$L_G^{\rm eff} = \frac{m^2(\dot{Z}^2/c^2 - |\nabla_{\boldsymbol{r}} Z|^2) + \dot{\alpha}^2/c^2 - |\nabla_{\boldsymbol{r}}\alpha|^2}{2m} + \frac{2M(Z-L)\dot{\alpha}}{\hbar} - m + \mathrm{O}(\nabla^3), \quad (8)$$

up to the quadratic order in derivatives. Here, $\nabla_{\boldsymbol{r}} = (\partial_x, \partial_y)$ is the derivative in the xy plane. The constant term m is the tension (the energy per unit area) of the static flat domain wall. The case in the massless limit $M \to 0$ was already obtained before [16].

The low-energy dynamics of Z and α derived from the Euler-Lagrange equation reads

$$\frac{m\ddot{Z}}{c^2} = \frac{2M\dot{\alpha}}{\hbar} + m\nabla_{\boldsymbol{r}}^2 Z, \qquad \frac{\ddot{\alpha}}{mc^2} = -\frac{2M\dot{Z}}{\hbar} + \frac{\nabla_{\boldsymbol{r}}^2\alpha}{m}. \quad (9)$$

In the massless limit $M \to 0$, the dynamics of Z and α are independent of each other, giving linear dispersions:

$$\omega = \pm c|\boldsymbol{k}|, \quad (10)$$

with the frequencies ω both for Z and α, and the wave number $\boldsymbol{k} = (k_x, k_y)$. Waves for Z and α independently propagate as a ripplon and a localized magnon in the vicinity of the domain wall.

As long as $M \neq 0$, the dynamics of Z and α couple to each other. There are four typical solutions of Eq (9),

$$Z_1^{\pm} = \frac{A_1^{\pm}}{m}\sin(\boldsymbol{k}\cdot\boldsymbol{r} \mp \omega_1 t + \delta_1^{\pm}),$$
$$\alpha_1^{\pm} = \pm A_1^{\pm}\cos(\boldsymbol{k}\cdot\boldsymbol{r} \mp \omega_1 t + \delta_1^{\pm}), \quad (11a)$$

$$Z_2^{\pm} = \frac{A_2^{\pm}}{m}\sin(\boldsymbol{k}\cdot\boldsymbol{r} \pm \omega_2 t + \delta_2^{\pm}),$$
$$\alpha_2^{\pm} = \pm A_2^{\pm}\cos(\boldsymbol{k}\cdot\boldsymbol{r} \pm \omega_2 t + \delta_2^{\pm}), \quad (11b)$$

where $A_{1,2}^{\pm} \in \mathbb{R}$ and $\delta_{1,2}^{\pm} \in \mathbb{R}$ are arbitrary constants. Waves of Z and α couple to each other and propagate as a coupled ripplon with dispersions

$$\omega_1 = \frac{\sqrt{M^2c^4 + \hbar^2 c^2 \boldsymbol{k}^2} - Mc^2}{\hbar} = \frac{\hbar \boldsymbol{k}^2}{2M} + O(\boldsymbol{k}^4),$$
$$\omega_2 = \frac{\sqrt{M^2c^4 + \hbar^2 c^2 \boldsymbol{k}^2} + Mc^2}{\hbar} = \frac{2Mc^2}{\hbar} + \frac{\hbar \boldsymbol{k}^2}{2M} + O(\boldsymbol{k}^4). \quad (12)$$

For the solutions of $(Z_1^\pm, \alpha_1^\pm)$, the coupled ripplons propagate in the direction parallel (for $+$ sign) and antiparallel (for $-$ sign) to $\boldsymbol{k}$ with a gapless quadratic dispersion ω_1 showing type-II NG modes [17]. Left and right panels of Fig. 1 show the schematic pictures of coupled ripplons for (Z_1^+, α_1^+) and (Z_1^-, α_1^-), respectively. In contrast to a quantized vortex in superfluids in which a Kelvin wave is a combination of two translational modes in real space, the coupled ripplon is a combination of the translational mode in real space and the phase mode of the internal degree of freedom. For the solutions of $(Z_2^\pm, \alpha_2^\pm)$, on the other hand, the coupled ripplons propagate in the opposite directions to $(Z_1^\pm, \alpha_1^\pm)$, respectively, with a gapped dispersion ω_2, and do not behave as NG modes. In the nonrelativistic limit $c \to \infty$, the gap in ω_2 diverges and the solutions $(Z_2^\pm, \alpha_2^\pm)$ disappear, and only $(Z_1^\pm, \alpha_1^\pm)$ for NG modes remain as solutions. In the massless limit $M \to 0$, the gap in ω_2 disappears and two dispersions $\omega_{1,2}$ become the same linear one: $\omega = c|k|$.

Linear response theory.—Dynamics of a ripplon can also be analyzed by the linear response theory. We consider the ansatz as the static domain-wall solution and its fluctuation: $u = u_0 + \delta u = u_0 + a_+ e^{i(\boldsymbol{k}\cdot\boldsymbol{r} - \omega t)} + a_-^* e^{-i(\boldsymbol{k}\cdot\boldsymbol{r} - \omega t)}$. Inserting this ansatz into the dynamical equation (5), we can obtain the Bogoliubov–de Gennes equation,

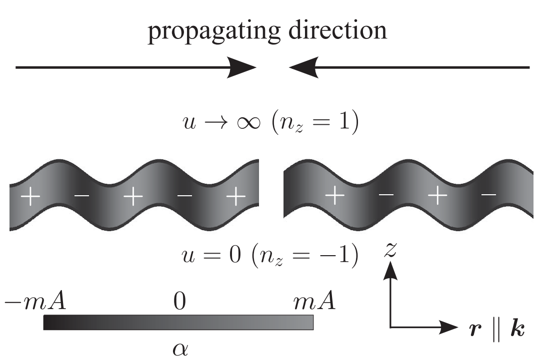

FIG. 1. Schematic pictures of coupled ripplons and their propagating directions for solutions (Z_1^+, α_1^+) (left) and (Z_1^-, α_1^-) (right). The middle shaded area shows the region of the domain wall $|u| \approx 1$ $(n_z \approx 0)$ and its tone shows the phase α of u [direction of (n_x, n_y)]. The $+$ and $-$ signs show the areas for $\alpha > 0$ and $\alpha < 0$, respectively. The vertical axis is the z axis and the horizontal axis shows the direction of the wave vector $\boldsymbol{k}$ in the xy plane. For left (right) figures, the coupled ripplon propagates in the right (left) direction. See the Supplemental Material for animations of their dynamics [11].

$$\left(\frac{\omega^2}{c^2} \pm \frac{2M\omega}{\hbar}\right) a_\pm$$
$$= \left\{(\boldsymbol{k}^2 - \partial_z^2) + \frac{4me^{2mz}\partial_z - m^2(3e^{2mz} - 1)}{1 + e^{2mz}}\right\} a_\pm + \mathrm{O}(a_\pm^2), \quad (13)$$

up to the linear order of $a_\pm$. The normalizable solution is $a_\pm \propto e^{mz}$ and corresponding ω takes the value $\omega_{1,2}$ shown in Eq. (12). In the massless limit $M \to 0$, $\omega_1 = \omega_2 = c|\boldsymbol{k}|$ gives the general solution $\delta u = e^{mz}\{g_1^+ e^{i(\boldsymbol{k}\cdot\boldsymbol{r} - c|\boldsymbol{k}|t)} + g_1^- e^{-i(\boldsymbol{k}\cdot\boldsymbol{r} - c|\boldsymbol{k}|t)} + g_2^+ e^{i(\boldsymbol{k}\cdot\boldsymbol{r} + c|\boldsymbol{k}|t)} + g_2^- e^{-i(\boldsymbol{k}\cdot\boldsymbol{r} + c|\boldsymbol{k}|t)}\}$ with arbitrary constants $g_{1,2}^\pm \in \mathbb{C}$. The localized magnon is obtained by taking $g_1^+ = -g_1^- = g_0 e^{i\delta}$ and $g_2^\pm = 0$ (parallel direction to $\boldsymbol{k}$), or $g_1^\pm = 0$ and $g_2^+ = -g_2^- = g_0 e^{i\delta}$ (antiparallel direction to $\boldsymbol{k}$) with $g_0, \delta \in \mathbb{R}$. The ripplon is obtained by taking $g_1 = g_2^* = g_0 e^{i\delta}$ and $g_3 = g_4 = 0$ (parallel direction to $\boldsymbol{k}$), or $g_1 = g_2 = 0$ and $g_3 = g_4^* = g_0 e^{i\delta}$ (antiparallel direction to $\boldsymbol{k}$). For $M \neq 0$ case, the solution is $\delta u_1 = e^{mz}\{g_1^+ e^{i(\boldsymbol{k}\cdot\boldsymbol{r} - \omega_1 t)} + g_1^- e^{-i(\boldsymbol{k}\cdot\boldsymbol{r} + \omega_1 t)}\}$ and $\delta u_2 = e^{mz}\{g_2^+ e^{i(\boldsymbol{k}\cdot\boldsymbol{r} + \omega_2 t)} + g_2^- e^{-i(\boldsymbol{k}\cdot\boldsymbol{r} - \omega_2 t)}\}$ with arbitrary constants $g_{1,2}^\pm \in \mathbb{C}$. $g_{1,2}^+ = iA_{1,2}^+ e^{i\delta_{1,2}^+}$ and $g_{1,2}^- = 0$ ($g_{1,2}^+ = 0$ and $g_{1,2}^- = -iA_{1,2} e^{i\delta_{1,2}^-}$) correspond to the coupled ripplon solution $(Z_{1,2}^+, \alpha_{1,2}^+)$ [$(Z_{1,2}^-, \alpha_{1,2}^-)$] in Eq. (11).

Commutation relation.—We obtain gapless and localized magnon and ripplon with linear dispersions, Eq. (10), only in the massless limit $M \to 0$ and the coupled ripplon with quadratic dispersion, Eq. (11a), with $M \neq 0$ in the vicinity of the domain wall. These modes are type-I (for $M \to 0$) and type-II (for $M > 0$) NG modes as a consequence of the spontaneous breaking of the U(1) symmetry for the phase rotation and the translational symmetry: $H_1 \simeq \mathrm{U}(1) \times \mathbb{R}^3 \to H_2 \simeq \mathbb{R}^2$ under the appearance of the domain wall. In both cases, our dispersions saturate the equality of the Nielsen-Chadha inequality [4]: $N_\mathrm{I} + 2N_\mathrm{II} \geq N_\mathrm{BG}$, where N_I, N_II, and N_BG are the total numbers of the type-I NG modes, the type-II NG modes, and broken generators (BG) which correspond to spontaneously broken symmetries. Recently, it has been shown in Refs. [7,8] that for internal symmetry the equality of the Nielsen-Chadha inequality is saturated as the Watanabe-Brauner's relation [6],

$$N_\mathrm{BG} - N_\mathrm{NG} = \frac{1}{2}\mathrm{rank}\rho,$$
$$\rho_{i,j} = \lim_{V \to \infty} \frac{1}{V} \int d^3\boldsymbol{x}(-i[\Omega_i, \Omega_j])\bigg|_{u=u_0}, \quad (14)$$

where $N_\mathrm{NG} = N_\mathrm{I} + N_\mathrm{II}$ is the number of NG modes, V is the volume of the system, Ω_i is the Noether's charge or a generator of broken symmetries, and $[\cdot, \cdot]$ is a commutator or the Poisson bracket in classical level. According to this relation, $N_\mathrm{BG} \neq N_\mathrm{NG}$ takes place when commutators of broken generators are nonvanishing. This relation has been proven for internal symmetries such as the Heisenberg ferromagnet and has been confirmed for space-time symmetries such as a quantized vortex in superfluid and a two-dimensional Skyrmion in ferromagnets [10]. In our case, on the other hand, broken generators consist of one internal symmetry and one spatial symmetry which intuitively commute because underlying symmetries are the direct product and are independent of each other, i.e., $H_1/H_2 \simeq \mathrm{U}(1)\times\mathbb{R}$. To check whether the relation, Eq. (14), also holds in our case or not, we directly calculate the commutation relation between symmetry generators of the internal U(1) phase rotation and the translation. Defining the momenta v conjugate to u as

$$v = \frac{\partial \mathcal{L}_G}{\partial \dot{u}} = \frac{\dot{u}^*}{c^2(1+|u|^2)^2} + \frac{iMu^*}{\hbar(1+|u|^2)}, \tag{15}$$

the Noether's charges for the phase rotation and the translation along z axis are obtained as

$$\Theta = \int dz J^0_\alpha, \qquad J^0_\alpha = iuv, \tag{16}$$

$$P = \int dz J^0_Z, \qquad J^0_Z = (\partial_z u)v, \tag{17}$$

respectively. The commutator between P and Θ can be calculated from $[u(z_1), v(z_2)] = i\delta(z_1 - z_2)$, to yield

$$\begin{aligned}
[P,\Theta] &= \int dz_1 \int dz_2 [J^0_Z(z_1), J^0_\alpha(z_2)] \\
&= i\int dz_1 \int dz_2 [\partial_{z_1} u(z_1)v(z_1), u(z_2)v(z_2)] \\
&= i\int dz_1 \int dz_2 \{\partial_{z_1} u(z_1)[v(z_1), u(z_2)]v(z_2) + u(z_2)\partial_{z_1}[u(z_1), v(z_2)]v(z_1)\} \\
&= \int dz_1 \int dz_2 \{\partial_{z_1} u(z_1)v(z_2) + u(z_2)\partial_{z_1} v(z_1)\}\delta(z_1 - z_2) \\
&= \int dz \partial_z [u(z)v(z)].
\end{aligned} \tag{18}$$

For the static domain-wall solution, the first term in Eq. (15) does not contribute to the commutator because of $\dot{u} = 0$. Consequently, the commutator becomes

$$\begin{aligned}
-i[P,\Theta] &= \frac{M}{\hbar}\int dz \partial_z \left(\frac{|u|^2}{1+|u|^2}\right) = \frac{M}{\hbar}\left[\frac{|u|^2}{1+|u|^2}\right]_{z=-\infty}^{z=+\infty} \\
&= \frac{2M}{\hbar}\left(\frac{1}{2}[1-n_z]_{z=-\infty}^{z=+\infty}\right) \equiv \frac{2MW}{\hbar}.
\end{aligned} \tag{19}$$

W is precisely the topological charge of the domain wall and is proportional to the tension of the domain wall [13,16] (see the Supplemental Material [11]). Evaluating this in the domain-wall background $u = u_0$, we find $W = 1$. As a result, two generators P and Θ do not commute as long as $M \neq 0$, giving $N_{\mathrm{BG}} - N_{\mathrm{NG}} = 1$ and one type-II NG mode, or commute in the massless limit $M \to 0$ giving two type-I NG modes, which is consistent with our result.

Conclusion.—In conclusion, we have considered NG modes excited on one flat domain wall in the $\mathbb{C}P^1$ models with the Ising potential. NG modes in the relativistic model are the localized magnon for the U(1) phase rotation and the translational ripplon which are independent of each other and have linear dispersions. In the nonrelativistic limit, on the other hand, there is one coupled ripplon with a quadratic dispersion as the combination of the localized magnon and the ripplon. We also find the coupled localized magnon and the ripplon in the interpolating model connecting the relativistic and nonrelativistic theories even though it has the Lorentz invariance. The numbers of NG modes saturate the equality of the Nielsen-Chadha inequality, and also satisfy the Watanabe-Brauner's relation in which the commutator between two generators of the internal phase mode and spatial translational mode gives the topological domain-wall charge.

Quantum effects on localized type-II NG modes remain as an important problem, which was studied for a vortex with non-Abelian localized modes [18].

The term $|u|^2/(1+|u|^2) = (1/2)(1-n_z)$ in Eq. (19) is known as the momentum map in symplectic geometry and the D-term in supersymmetric gauge theory. Therefore, our model can be extended to the $\mathbb{C}P^n$ model, Grassmann σ model [19,20], σ models on more general Kähler target manifolds, and non-Abelian gauge theories. A domain wall in two-component Bose-Einstein condensates has a different structure of NG modes [21], although there are also translational and internal U(1) zero modes [22]. This may be because the U(1) zero mode is non-normalizable in their case. If one couples a gauge field, their model reduces to ours in the strong gauge coupling limit (see the Supplemental Material [11]), with the internal U(1) mode becoming normalizable.

We thank H. Takeuchi for useful discussions and H. Watanabe for explaining their results. We also thank the anonymous referees for the helpful suggestions and comments. This work is supported in part by Grant-in-Aid for Scientific Research [Grants No. 26870295 (M. K.) and No. 25400268 (M. N.]) and the work of M. N. is also supported in part by the "Topological Quantum Phenomena" Grant-in-Aid for Scientific Research on Innovative Areas (No. 25103720) from the Ministry of Education, Culture, Sports, Science, and Technology (MEXT) of Japan.

[1] S. R. Coleman and J. Mandula, Phys. Rev. **159**, 1251 (1967).

[2] If we relax one of the assumptions that the Coleman-Mandula theorem is based on, supersymmetry (which is a fermionic symmetry) can combine space-time symmetry and internal symmetry, as shown by R. Haag, J. T. Łopuszański, and M. Zohnius in Nucl. Phys. **B88**, 257 (1975). Namely, the supersymmetry algebra $\{Q^I_\alpha, \bar{Q}^J_\beta\} \sim \delta^{IJ} P_\mu \sigma^\mu_{\alpha\beta}$ and $\{Q^I_\alpha, Q^J_\beta\} = \epsilon_{\alpha\beta} C^{IJ}$ contains the translational operator $\boldsymbol{P}$ and the indices $\boldsymbol{I}$, $\boldsymbol{J}$ for an internal symmetry, where C^{IJ} is a central extension.

[3] This is similar to supersymmetry algebra in the presence of Bogomol'nyi-Prasad-Sommerfield (BPS) kinks, as shown in E. Witten and D. I. Olive, Phys. Lett. **78B**, 97 (1978). More precisely, $\boldsymbol{W}$ can be a tensorial charge rather than a central charge for extended objects as supersymmetry algebra with BPS extended topological solitons and $\boldsymbol{p}$-branes in supergravity and string theory; See G. R. Dvali and M. A. Shifman, Nucl. Phys. **B504**, 127 (1997); Phys. Lett. B **396**, 64 (1997); , **407**, 452(E) (1997); A. Gorsky and M. A. Shifman, Phys. Rev. D **61**, 085001 (2000), for BPS extended topological solitons; J. A. de Azcarraga, J. P. Gauntlett, J. M. Izquierdo, and P. K. Townsend, Phys. Rev. Lett. **63**, 2443 (1989); E. R. C. Abraham and P. K. Townsend, Nucl. Phys. **B351**, 313 (1991); P. K. Townsend, in *The World in Eleven Dimensions: Supergravity, Supermembranes, and M-Theory*, Series in High Energy Physics, Cosmology, and Gravitation, edited by M. J. Duff (CRC Press, Boca Raton, FL, 1999), pp. 375–389, for $\boldsymbol{p}$-branes.

[4] H. B. Nielsen and S. Chadha, Nucl. Phys. **B105**, 445 (1976).

[5] Y. Nambu, J. Stat. Phys. **115**, 7 (2004).

[6] H. Watanabe and T. Brauner, Phys. Rev. D **84**, 125013 (2011).

[7] H. Watanabe and H. Murayama, Phys. Rev. Lett. **108**, 251602 (2012).

[8] Y. Hidaka, Phys. Rev. Lett. **110**, 091601 (2013).

[9] M. Kobayashi and M. Nitta, Prog. Theor. Exp. Phys. 021B01 (2014).

[10] H. Watanabe and H. Murayama, Phys. Rev. Lett. **112**, 191804 (2014).

[11] See the Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevLett.113.120403 for mathematical details and animations of coupled ripplons.

[12] The massless limit $M \to 0$ is often called the ultrarelativistic limit and $\mathcal{L}_{\rm rel}$ may have to be called an ultrarelativistic model. In Refs. [4–10], however, similar ultrarelativistic models with only a second time derivative term are referred as relativistic models. We therefore refer to $\mathcal{L}_{\rm rel}$ as a relativistic model too.

[13] E. R. C. Abraham and P. K. Townsend, Phys. Lett. B **291**, 85 (1992); **295**, 225 (1992).

[14] In the existence of the domain wall, the three-dimensional SO(3) rotational symmetry in space is also broken to the two-dimensional SO(2) rotational symmetry along the z axis. The breaking rotational symmetry along the direction perpendicular to the z axis corresponding to $\mathrm{SO}(3)/\mathrm{SO}(2) \simeq S^2$, however, does not contribute as additional Nambu-Goldstone modes, because the rotational modes can be constructed from the local translational mode, as discussed in E. A. Ivanov and V. I. Ogievetskii, Theor. Math. Phys. **25**, 1050 (1975); I. Low and A. V. Manohar, Phys. Rev. Lett. **88**, 101602 (2002).

[15] N. S. Manton, Phys. Lett. **110B**, 54 (1982); M. Eto, Y. Isozumi, M. Nitta, K. Ohashi, and N. Sakai, Phys. Rev. D **73**, 125008 (2006).

[16] M. Arai, M. Naganuma, M. Nitta, and N. Sakai, Nucl. Phys. **B652**, 35 (2003); M. Nitta, Phys. Rev. D **86**, 125004 (2012).

[17] Here, we note that the term of type I or II is defined in Ref. [4], that the exponent of the dispersion is odd or even, whereas that of type A or B is defined in Ref. [7] from the commutation relations of the corresponding broken generators. In our case, type I (II) and type A (B) coincide, and we use the terminology type I or II.

[18] M. Nitta, S. Uchino, and W. Vinci, arXiv:1311.5408 [J. High Energy Phys. (to be published)].

[19] Y. Isozumi, M. Nitta, K. Ohashi, and N. Sakai, Phys. Rev. Lett. **93**, 161601 (2004); Phys. Rev. D **70**, 125014 (2004); **71**, 065018 (2005); M. Eto, Y. Isozumi, M. Nitta, K. Ohashi, K. Ohta, and N. Sakai, Phys. Rev. D **71**, 125006 (2005).

[20] M. Eto, Y. Isozumi, M. Nitta, K. Ohashi, and N. Sakai, J. Phys. A **39**, R315 (2006).

[21] H. Takeuchi and K. Kasamatsu, Phys. Rev. A **88**, 043612 (2013).

[22] The dispersion relation is $\omega \sim k^{3/2}$ for large k while it is quadratic for small momenta with a finite system size. See Ref. [21] and H. Watanabe and H. Murayama, Phys. Rev. D **89**, 101701 (2014); D. A. Takahashi and M. Nitta, arXiv:1404.7696.